

Topic : Viscosity

Type of Questions

Comprehension ('-1' negative marking) Q.1 to Q.3

(3 marks, 3 min.)

M.M., Min.

[9, 9]

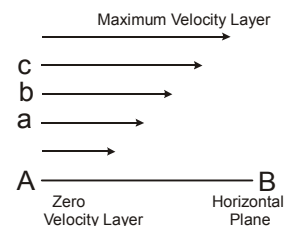
COMPREHENSION

VISCOSITY

When a solid body slides over another solid body, a frictional-force begins to act between them. This force opposes the relative motion of the bodies. Similarly, when a layer of a liquid slides over another layer of the same liquid, a frictional-force acts between them which opposes the relative motion between the layers. This force is called 'internal frictional-force'.

Suppose a liquid is flowing in streamlined motion on a fixed horizontal surface AB (Fig.). The layer of the liquid which is in contact with the surface is at rest due to adhesive forces between the liquid and the surface, while the velocity of other layers increases with distance from the fixed surface. In the Fig., the lengths of the arrows represent the increasing velocity of the layers. Thus there is a relative motion between adjacent layers of the liquid. Let us consider three parallel layers a, b and c. Their velocities are in the increasing order. The layer a tends to retard the layer b, while b tends to retard c.

Thus each layer tends to decrease the velocity of the layer above it. Similarly, each layer tends to increase the velocity of the layer below it. This means that in between any two layers of the liquid, internal tangential forces act which try to destroy the relative motion between the layers. These forces are called 'viscous forces'. If the flow of the liquid is to be maintained, an external force must be applied to overcome the dragging viscous forces. In the absence of the external force, the viscous forces would soon bring the liquid to rest. **The property of the liquid by virtue of which it opposes the relative motion between its adjacent layers is known as 'viscosity'.**



The property of viscosity is seen in the following examples :

- (i) A stirred liquid, when left, comes to rest on account of viscosity. Thicker liquids like honey, coaltar, glycerine, etc. have a larger viscosity than thinner ones like water. If we pour coaltar and water on a table, the coaltar will stop soon while the water will flow upto quite a large distance.
- (ii) If we pour water and honey in separate funnels, water comes out readily from the hole in the funnel while honey takes enough time to do so. This is because honey is much more viscous than water. As honey tends to flow down under gravity, the relative motion between its layers is opposed strongly.
- (iii) We can walk fast in air, but not in water. The reason is again viscosity which is very small for air but comparatively much larger for water.

Viscosity comes into play only when there is a relative motion between the layers of the same material. This is why it does not act in solids.

VELOCITY GRADIENT AND COEFFICIENT OF VISCOSITY

The property of a liquid by virtue of which an opposing force (internal friction) comes into play whenever there is a relative motion between the different layers of the liquid is called viscosity. Consider a flow of a liquid over the horizontal solid surface as shown in fig. Let us consider two layers AB and CD moving with velocities

\vec{v} and $\vec{v} + d\vec{v}$ at a distance x and $(x + dx)$ respectively from the fixed solid surface.

According to Newton, the viscous drag or back ward force (F) between these layers depends.

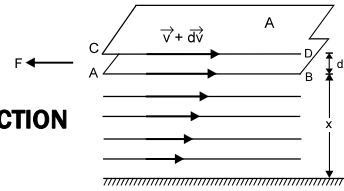
- (i) directly proportional to the area (A) of the layer and (ii) directly proportional to the velocity gradient

$\left(\frac{dv}{dx}\right)$ between the layers.

i.e. $F \propto A \frac{dv}{dx}$ or $F = -\eta A \frac{dv}{dx} \dots(1)$



η is called **Coefficient of viscosity**. Negative sign shows that the direction of viscous drag (F) is just opposite to the direction of the motion of the liquid.



SIMILARITIES AND DIFFERENCES BETWEEN VISCOSITY AND SOLID FRICTION

Similarities

Viscosity and solid friction are similar as

- Both oppose relative motion. Whereas viscosity opposes the relative motion between two adjacent liquid layers, solid friction opposes the relative motion between two solid layers.
- Both come into play, whenever there is relative motion between layers of liquid or solid surfaces as the case may be.
- Both are due to molecular attractions.

Differences between them →

	Viscosity	Solid Friction
(i)	Viscosity (or viscous drag) between layers of liquid is directly proportional to the area of the liquid layers.	Friction between two solids is independent of the area of solid surfaces in contact.
(ii)	Viscous drag is proportional to the relative velocity between two layers of liquid.	Friction is independent of the relative velocity between two surfaces.
(iii)	Viscous drag is independent of normal reaction between two layers of liquid in contact.	Friction is directly proportional to the normal reaction between two surfaces in contact.

EFFECT OF TEMPERATURE ON THE VISCOSITY

The viscosity of liquids decrease with increase in temperature and increase with the decrease in temperature.

That is, $\eta \propto \frac{1}{\sqrt{T}}$. On the other hand, the value of viscosity of gases increases with the increase in temperature and vice-versa. That is, $\eta \propto \sqrt{T}$ (This takes into account the diffusion of the gases).

SOME APPLICATIONS OF VISCOSITY

Knowledge of viscosity of various liquids and gases have been put to use in daily life. Some applications of its knowledge are discussed as under →

- As the viscosity of liquids vary with temperature, proper choice of lubricant is made depending upon season. Coaltar used for making of roads is heated to reduce its viscosity so that it can be easily laid on the road.
- Liquids of high viscosity are used in shock absorbers and buffers at railway stations.
- The phenomenon of viscosity of air and liquid is used to damp the motion of some instruments like galvanometer

UNITS OF COEFFICIENT OF VISCOSITY

From the above formula, we have $\eta = \frac{F}{A(\Delta v_x / \Delta z)}$

$$\therefore \text{dimensions of } \eta = \frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]} = \frac{[MLT^{-2}]}{[L^2T^{-1}]} = [ML^{-1}T^{-1}]$$

Its unit is kg/(meter-second)

In C.G.S. system, the unit of coefficient of viscosity is dyne s cm⁻² and is called poise. In SI the unit of coefficient of viscosity is N sm⁻² and is called decapoise.

1 decapoise = 1 N sm⁻² = (10⁵ dyne) × s × (10² cm)⁻² = 10 dyne s cm⁻² = 10 poise

Illus. 1.

A man is rowing a boat of mass m with a constant velocity ' v_0 ' in a river the contact area of boat is 'A' and coefficient of viscosity is η . The depth of river is 'D'. Find the force required to row the boat. Assume that velocity gradient is constant.

Sol.

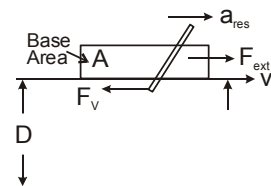
$$F_{\text{ext}} - F_v = m a_{\text{res}}$$

As boat moves with constant velocity $a_{\text{res}} = 0$

$$F_{\text{ext}} = F_v$$

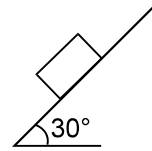
But $F_v = \eta A \frac{dv}{dz}$, but $\frac{dv}{dz} = \frac{v_0 - 0}{D} = \frac{v_0}{D}$

then $F_{\text{ext}} = F_v = \frac{\eta A v_0}{D}$



Illus. 2.

A cubical block (of side 2m) of mass 20 kg slides on inclined plane lubricated with the oil of viscosity $\eta = 10^{-1}$ poise with constant velocity of 10 m/sec. ($g = 10 \text{ m/sec}^2$)
find out the thickness of layer of liquid. (10^{-1} poise = 10^{-2} Nsm^2)

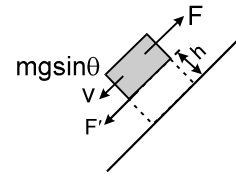


Sol. $F = F' = \eta A \frac{dv}{dz} = mg \sin \theta \quad \frac{dv}{dz} = \frac{v}{h}$

$$20 \times 10 \times \sin 30^\circ = \eta \times 4 \times \frac{10}{h}$$

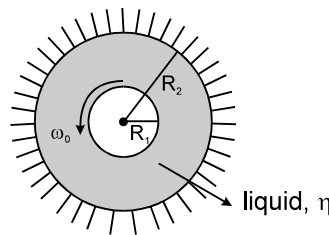
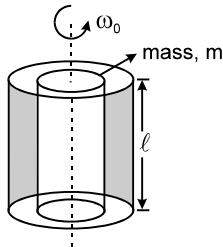
$$h = \frac{40 \times 10^{-2}}{100} = [\eta = 10^{-1} \text{ poise} = 10^{-2} \text{ N-sec-m}^{-2}]$$

$$= 4 \times 10^{-3} \text{ m} = 4 \text{ mm}$$



Now Answer the questions below :

- A metal square plate of 10 cm side rests on a 2 mm thick castor oil layer. Calculate the horizontal force needed to move the plate with speed 3 cm s^{-1} : (Coefficient of viscosity of castor oil is 15 poise.)
(A) $2.25 \times 10^{-2} \text{ N}$ (B*) $2.25 \times 10^{-1} \text{ N}$ (C) $2.25 \times 10^{-3} \text{ N}$ (D) $2.25 \times 10^{-4} \text{ N}$
- A man starts rowing his stationary cuboidal boat of base area $A = 10 \text{ m}^2$. The driving force on the boat due to rowing is 100 N in the direction of motion. Find the maximum velocity that the boat can achieve. Also find the time in which he will attain half of this maximum velocity. [Take coefficient of viscosity of water = 15 poise] The depth of the lake is 10 m and the combined mass of man and the boat to be 150 kg. ($u = 0$, velocity gradient uniform)
- As per the shown figure the central solid cylinder starts with initial angular velocity ω_0 . Find out the time after which the angular velocity becomes half. (Velocity gradient uniform)



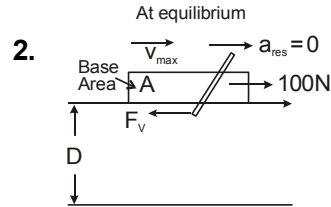
Answers Key

DPP NO. - 91

- (B) 2. $\frac{200}{3} \text{ m/s}, t = 100 \ln 2 \text{ seconds}$
- $t = \frac{m (R_2 - R_1) \ln 2}{4 \pi \eta \ell R_1}$

Hint & Solutions

DPP NO. - 91



At equilibrium

$$F_{\text{ext}} = F_v$$

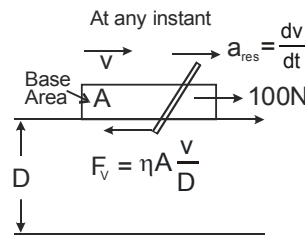
$$F_{\text{ext}} = 100 \text{ N and } F_v = \eta A \frac{v_{\max}}{D} = 1.5 \times 10 \times \frac{v_{\max}}{D}$$

$$\therefore 100 = 1.5 v_{\max}$$

$$\Rightarrow v_{\max} = \frac{200}{3} \text{ m/s}$$

At any instant

$$F_{\text{ext}} - F_v = m \frac{dv}{dt}$$



$$\Rightarrow 100 - \eta A \frac{v}{D} = m \frac{dv}{dt}$$

$$\therefore 100 - 1.5 v = 150 \frac{dv}{dt}$$

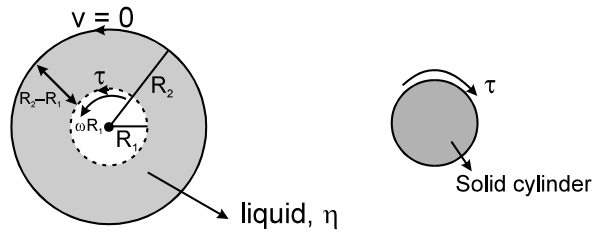
$$\text{from (1)} \int_0^t dt = \int_0^{\frac{v_{\max}}{2}} 150 \frac{dv}{100 - 1.5v}$$

$$\Rightarrow t = 150 \int_0^{100/3} \frac{\ln(100 - 1.5v)}{-1.5}$$

$$\Rightarrow t = 100 \ln 2 \text{ seconds}$$

3. $F = \eta A \frac{dv}{dz}$, where $\frac{dv}{dz} = \frac{\omega R_1 - 0}{R_2 - R_1}$

$$F = \eta \frac{2\pi R_1 \ell \omega R_1}{R_2 - R_1}$$



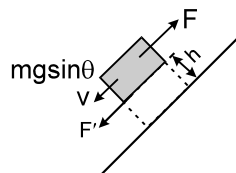
and $\tau = FR_1 = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$

$$I \alpha = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$$

$$\Rightarrow \frac{MR_1^2}{2} \left(-\frac{d\omega}{dt} \right) = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$$

$$\text{or } - \int_{\omega_0}^{\omega_0/2} \frac{d\omega}{\omega} = \frac{4\pi\eta R_1 \ell}{m(R_2 - R_1)} \int_0^t dt$$

$$\Rightarrow t = \frac{m(R_2 - R_1) \ln 2}{4\pi\eta \ell R_1}$$



$$h = \frac{40 \times 10^{-2}}{100} \quad [\eta = 10^{-1} \text{ poise} = 10^{-2} \text{ N-sec-m}^{-2}]$$

$$= 4 \times 10^{-3} \text{ m} = 4 \text{ mm}$$